COMMENT ON CURRENT VERSION OF A. P. KHOKHLOV'S ARTICLE "THE THEORY OF RESONANCE INTERACTION OF TOLLMIEN-SCHLICHTING WAVES"

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The study referred to can be considered as one of the modified theories of weakly nonlinear critical layers for three-dimensional disturbances. Starting from the system of equations of the three-tier free interaction scheme (which is itself obtained as a result of certain assumptions concerning the orders of dependent and independent variables and the representations of solutions of the Navier–Stokes equation in the form of special series in negative powers of the Reynolds number), the author introduces, within the asymptotic scheme indicated, new expansions in the shear unperturbed solution $u = y$, $v = w = 0$. Investigating spatially periodic disturbances with wave vector components α, β, and assuming that the quantity $\varepsilon = c^{-1}$, where c is the phase velocity, is a small parameter, from the dispersion relations of linear stability theory one can directly obtain the length scale $\Delta x \sim \alpha^{-1} \sim \varepsilon$, adopted in the study. The nonstationary critical layer determining the time scale $\Delta t \sim \varepsilon^{2/3}$, which is "slow" in comparison with the characteristic time $\Delta t \sim \varepsilon^2$ of oscillations in a linear Tollmien–Schlichting wave. In fact, the expansion parameter of the solution sought is $\varepsilon^{2/3}$, which is guaranteed by the trivial estimates given above (the terms differing from the first approximation by a quantity of the order of $\varepsilon^{2/3}$ are included in the first approximation itself, therefore the second approximation for all functions considered with respect to the first approximation is not of order $\varepsilon^{2/3}$, but of order $\varepsilon^{4/3}$; the orders of smallness of the following approximations increase by the quantity mentioned $\varepsilon^{2/3}$).

Less trivial is the choice of the first approximation amplitude. We denote the disturbance amplitude of the longitudinal velocity components in the critical layer by $\delta = \varepsilon^n$. Though it is also stated in the study reviewed that the order of magnitude of slow time uniquely provides the characteristic amplitude, an estimate for δ is established by the additional requirement of appearance of nonlinear terms in the fourth approximation equations. Indeed, the first and fourth approximations differ by a quantity of order $\varepsilon^{8/3}$ (for all functions and in all expansions). From the equality condition of the orders of the terms $\partial\hat{u}_4/\partial t_1$ and $\hat{u}_1\partial\hat{u}_1/\partial x$ in the critical layer one obtains $\delta = \varepsilon^3$, which was also adopted in the study reviewed.

The basic idea of the study consists of determining the dependencies on slow time of the first approximation from the solvability conditions of the equation appearing in the problem of determining the pressure and the functions contained in the fourth approximation. The arguments provided above, justifying the choice of the amplitude, are absent in the new version of the study reviewed. It is not quite clear whether different methods of estimating amplitudes can be suggested. Obviously, the study reviewed would have gained substantially in the case of deciphering the form in which the solution is sought. Reading it in the form presented, the disturbance evolution properties described are perceived as a result of formal substitution into three-dimensional expansions of interaction equations of some form, leading to compatible properties of a class of problems.

The problem touched upon of selecting δ is important, since when $\delta \ll \varepsilon^3$ the theory considered must transform to linear stability theory, for which, when $\alpha \gg 1$, $\beta \gg 1$, we have

$$\omega = \alpha(\alpha^2 + \beta^2)^{1/2} + O(1).$$

Here the term O(1), absent in the equation for ω given by the author, contains an imaginary part, determining the growth rate of linear oscillations. This implies that in the problem, along with the times $t_0 \sim \varepsilon^{-2}t$, $t_1 \sim \varepsilon^{-2/3}t$, there is also a time $t_2 \sim t$. Account of the time t_2 can change the equations for approximations exceeding fourth order (we recall that all expansions in the study reviewed are carried out in the parameter $\varepsilon^{2/3}$). Therefore, the suspicion arises that not including in the asymptotic series the dependence on the time t_2

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may not only lead to an incorrect growth rate of disturbances within the linearized problems, but may also distort the evolution amplitude as a function of time t_1 in the nonlinear regime.

Thus, the lack of clarity in presenting the form of the solution, marked by the present referees in commenting on the original version of the study reviewed, must be understood not simply as a wish for an improved style of presentation, but an intrinsically vague formulation of the problem considered. From the revised version of the article it is obvious that the referees' comments have so far not been grasped by the author.

Nevertheless, A. P. Khokhlov's article "The theory of resonance interaction of Tollmien–Schlichting waves" deserves publication in Prikladnaya Mekhika Tekhicheskaya Fizika, though in a form presented following suitable corrections. As follows from the discussion above, however, the problem of whether the results presented in it are asymptotically correct is still open.

EFFECT OF POLYDISPERSION ON SOUND PROPAGATION IN GAS MIXTURES WITH VAPOR AND LIQUID DROPS

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The propagation of nonstationary low-amplitude disturbances in heterogeneous gas mixtures with vapor and liquid drops is one of the current problems of wave dynamics of two-phase systems. Such heterogeneous media are basic working units in energy devices, apparatus of chemical technology, and other devices of contemporary technology. In these cases, to control the flow of various technological processes one often uses calculations and measurements of propagation and absorption rates of acoustic waves. Therefore, investigations of the effects of various physicochemical transformations on the character of disturbance propagation in two-phase gas–drop systems are very valuable.

Despite the number of published studies, the propagation of acoustic waves in vapor–gas–drop systems in the presence of interphase in the presence of mass exchange has so far not been investigated in sufficient detail. Most of the studies in acoustics of gas–vapor–drop media were devoted to the study of propagation of low-intensity waves in multiply disperse systems [1-12]. A number of aspects of the effect of polydispersion on propagation of acoustic disturbances in gas suspensions in the absence of mass exchange was treated earlier in [1, 13]. The problem of sound propagation in polydisperse vapor–gas–drop mixtures has practically not been investigated. In the present study we investigate for the first time the effect of polydispersion on the propagation of low-intensity waves in vapor–gas–drop systems, including effects of nonequilibrium phase transformations.

1. Consider the one-dimensional motion of a polydisperse vapor–gas–drop mixture in an acoustic field, when the disturbances in mixture parameters are small. The basic characteristics of this suspension are the following parameters:

$$n=\int_{a_{\min}}^{a_{\max}} N(a)\,da,\quad \alpha_2=\int_{a_{\min}}^{a_{\max}} \frac{4}{3}\pi a^3 N(a)\,da,\quad \alpha_2+\alpha_1=1,$$

$$\rho_1=\alpha_1\rho_1^0,\quad \rho_2=\alpha_2\rho_2^0=\int_{a_{\min}}^{a_{\max}} m_2(a)\,N(a)\,da,\quad m_2=\frac{4}{3}\pi a^3\rho_2^0,$$

$$m=\rho_{20}/\rho_{10},\ k_j=\rho_{j0}/\rho_{10},\ j=V,\ G,\ k_V+k_G=1.$$

Here N(a) is the size distribution function of drops in the suspension with minimum $a_{\min}$ and maximum $a_{\max}$ drop radii, n, α_i, ρ_i^0, ρ_i are the total number of particles per unit volume, the bulk content, and the true and mean densities of the gas phase (i = 1) and of particles (i = 2), m_2 and m are the mass of a single drop and the initial mass content of drops, k_j is the initial concentration of the vapor (j = V) and gas (j = G) components of the gas phase, and

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